

Math 1113 Final Exam Review

Rouhani

This review should only be used as a guide for preparation for the final exam. In addition you should study your notes, homework, and quizzes.

Section 1

Suppose graph of f is given. Describe the following graphs relative to graph of f (#1-3).

1. $y = \frac{1}{2}f(x + 2)$

ans: Vertical shrink by a factor of $\frac{1}{2}$ units, then horizontal shift of 2 units to the left

2. $y = f(\frac{1}{5}x) - 1$

ans: Horizontal stretch of 5 units, then vertical shift of one unit down.

3. $y = -5f(x) + 1$

ans: Vertical stretch of 5 units, reflection along x-axis, then vertical shift of one unit up.

4. Identify each translation from the function $f(x) = x^2$ to $g(x)$.

a. $g(x) = -5(3x - 2)^2 - 2$

ans: Horizontal shrink by a factor of $\frac{1}{3}$, horizontal shift of 2 units to the right, vertical stretch by a factor of 5 units, reflection along the x-axis, finally vertical translation of 2 units down.

b. $g(x) = \frac{1}{4}x^2 + 3$

ans: Vertical shrink by a factor of $\frac{1}{4}$, then vertical shift of 3 units up.

c. $g(x) = -2x^2 + 1$

ans; Vertical stretch of 2 units, reflection along x-axis, then vertical translation of 1 unit up.

Find the domain of each function.

5. $f(x) = \frac{2x}{5x - 10}$

ans: $(-\infty, 2), (2, \infty)$

6. $f(x) = 3x^3 + 5x^2 - 1$

ans: $(-\infty, \infty)$

7. $f(x) = \frac{3x}{3x^2 - 7x - 6}$ **ans:** $(-\infty, -\frac{2}{3}), (-\frac{2}{3}, 3), (3, \infty)$

Evaluate the indicated functions, where $f(x) = x^2 - 2x + 1$ and $g(x) = x - 3$.

8. $(f + g)(2)$ **ans:** 0

9. $\left(\frac{f}{g}\right)(-4)$ **ans:** $-\frac{25}{7}$

Find $f \circ g$ for the given functions f and g .

10. $f(x) = \sqrt{x+4}$, $g(x) = \frac{1}{x}$ **ans:** $f \circ g = \frac{\sqrt{x+4x^2}}{x}$

11. $f(x) = -x^3 - 7$, $g(x) = x + 1$ **ans:** $f \circ g = -x^3 - 3x^2 - 3x - 8$

Find inverse of each function.

12. $f(x) = 5x - 2$ **ans:** $f^{-1}(x) = \frac{x+2}{5}$

13. $f(x) = \frac{x+3}{x-2}$ **ans:** $f^{-1}(x) = \frac{3+2x}{x-1}$

14. Given $f(x) = \sqrt{x-5}$ and $g(x) = \sqrt{x+3}$ find $f(g(x))$ then determine domain of $f \circ g$.

ans: $f(g(x)) = \sqrt{\sqrt{x+3}-5}$ Domain = $[22, \infty)$

15. Given $f(x) = \sqrt{x}$, $g(x) = x - 2$ find $f(g(x))$ then determine domain of $f \circ g$.

ans: $f(g(x)) = \sqrt{x-2}$ Domain = $[2, \infty)$

Also you should know how to:

- a. Identify intervals that a given function is constant, increasing, or decreasing.
- c. Identify the domain and range of a graph of a function.
- d. Find inverse of a function given its graph.
- e. Use the horizontal and vertical line tests.
- f. Find equation of a function given its graph.
- g. Distinguish between vertical and horizontal stretch and shrink.

Section 2

1. Find the intercepts of the graph of each function.

a. $f(x) = x^2 - 9x + 18$ **ans:**x-intercept: (3,0), (6,0) y-intercept: (0,18)

b. $f(x) = (x - 3)^2 - 2$ **ans:**x-intercept: $(3 \pm \sqrt{2}, 0)$ y-intercept:(0,7)

2. Identify any vertical and horizontal asymptotes of the following functions.

a. $f(x) = \frac{3x^2 + x - 5}{x^2 + 1}$ **ans:** vertical asymptote: none ; horizontal asymptote: $y = 3$

b. $f(x) = \frac{2x^2}{x + 1}$ **ans:**vertical asymptote: $x = -1$; horizontal asymptote: none

c. $f(x) = \frac{2}{(x - 3)^2}$ **ans:**vertical asymptote: $x = 3$; horizontal asymptote: $y = 0$

3. Find intercepts of each rational function.

a. $f(x) = \frac{2x^2}{x^2 - 1}$ **ans:** x-int = 0, y-int = 0

b. $f(x) = \frac{3x^2 - 2x - 5}{x^2 + 1}$ **ans:** x-int = $-1, \frac{5}{3}$; y-int = -5

Also you should know:

- The difference between rational and polynomial functions.
- What are zeros of a polynomial functions, and how to find them. How to find multiplicity of a zero.
- Even and odd functions
- How to find average rate of change of functions between two given points.

Section 3

Solve for x (# 1-8).

1. $2^{x+2} = 47$ **ans:** $x \approx 3.55$

2. $\ln(x - 1) = 1$ **ans:** $x = 3.71$

3. $e^{2x} = 4$ **ans:** $x \approx 0.69$

4. $e^{\ln(5x+2)} = 7$ **ans:** $x = 1$
5. $\log_3 x + \log_3(x^2 - 8) = \log_3 8x$ **ans:** $x = 4$
6. $\log(x - 3) = 2$ **ans:** $x = 103$
7. $6e^{2x} + 5 = 2 + 3e^{2x}$ **ans:** No solution
8. $4x^2(2^x) - 9(2^x) = 0$ **ans:** $x = \pm \frac{3}{2}$

Write the expression as a logarithm of a single quantity.

9. $\ln x - 2[\ln(x + 2) + \ln(x - 2)]$ **ans:** $\ln \frac{x}{(x^2 - 4)^2}$
10. $\log x - 3 \log(x - 1)$ **ans:** $\log \frac{x}{(x - 1)^3}$

11. Find domain of $f(x) = \ln(2 - x) + 1$. **ans:** $(-\infty, 2)$
12. Find domain of $f(x) = \log x - 2$ **ans:** $x = (0, \infty)$
13. Find domain of $f(x) = \log_3(3x - 1) + \sqrt{1 - x}$ **ans:** $(\frac{1}{3}, 1]$

Use your graphing utility to sketch the graph of each function, then state the domain, range, intercepts and asymptote of f .

14. $f(x) = 2^{x+1} + 2$
ans: Domain: $(-\infty, \infty)$; Range: $(2, \infty)$; x-int: *none* ; y-int: $(0, 4)$; asymptote:
 $y = 2$
15. $f(x) = -\log(x + 2)$
ans: Domain: $(-2, \infty)$; Range: $(-\infty, \infty)$; x-int: $(-1, 0)$; y-int: $(0, -0.3)$; asymptote:
 $x = -2$
16. How long will it take for \$2500 to triple if it is invested in a savings account that pays 4.5% interest compounded continuously?
ans: approx 24 years
17. Suppose \$3000 is invested into an account paying 6% interest compounded quarterly. Find the balance in the account after 5 years.

ans: \$4040.56

18. The population of a town is modeled by $P = 12.62e^{0.0118t}$ where $t = 0$ represents the year 2000. According to this model, when will the population reach 18,000?

ans: year 2615

19. Radioactive strontium decays according to the function $y = y_0e^{-0.0239t}$, where t is time in years.

- a. If an initial sample contains $y_0 = 5$ grams of radioactive strontium, how many grams will be present after 60 years?
- b. What is the half-life of radioactive strontium?

ans: a. 1.19 grams b. 29 years

20. The half-life of a certain radioactive material is 1200 years.

- a. Find the decay constant k . **ans:** -0.000577
- b. What percent of material will remain after 135 years. **ans:** 92%

Section 4

For problems 1-3 you are given the rate of rotation of a wheel as well as its radius. In each case, determine the following:

- a. *the angular speed, in units of radians/sec.*
- b. *the linear speed, in units of cm/sec, of a point on the circumference of the wheel*

1. 6 revolutions/sec; $r = 12$ cm **ans:** a. 12π rad/sec b. 144π cm/sec

2. 1080° /sec; $r = 25$ cm **ans:** a. 6π rad/sec b. 150π cm/sec

3. 500 rpm; $r = 45$ cm **ans:** a. $\frac{50\pi}{3}$ rad/sec b. 750π cm/sec

4. Find the values of the six trigonometric functions of the angle in standard position with the terminal side passing through the point $(1, -3)$.

ans: $\sin \theta = -\frac{3\sqrt{10}}{10}$ $\cos \theta = \frac{\sqrt{10}}{10}$ $\tan \theta = -3$ $\cot \theta = -\frac{1}{3}$ $\sec \theta = \sqrt{10}$ $\csc \theta = -\frac{\sqrt{10}}{3}$

5. If θ is an acute angle in standard position and $\sin \theta = \frac{3}{5}$, find $\sec \theta$.

ans: $\frac{5}{4}$

6. Find amplitude and period of the function $f(x) = -4 \sin \frac{4\pi}{3}$.

ans: amp = 4, period = $\frac{3}{2}$

7. Determine an equation for a cosine curve that has amplitude of 3, period of π , and phase shift of $\frac{\pi}{4}$ to the left.

ans: $y = 3 \cos(2x + \frac{\pi}{2})$.

8. Determine an equation for a sine curve that has amplitude of 2, period of 2, and phase shift of $\frac{2}{3}$ to the right.

ans: $y = 2 \sin(\pi x - \frac{2\pi}{3})$

9. Given that $\tan \theta = -\sqrt{2}$, $\sin \theta > 0$; find $\sec \theta$.

ans: $-\sqrt{3}$

10. A tree casts a shadow of 8.55 ft when the angle of elevation of the sun is 55.3° . Find the height of the tree.

ans: 12.3 feet

11. Find x-intercepts of each trigonometric function in the specified interval.

$y = 3 \cos(4\pi x - \frac{\pi}{4})$, $[0, \frac{1}{2}]$

ans: $(\frac{3}{16}, 0), (\frac{7}{16}, 0)$

$y = \sin \frac{2\pi}{3}$, $[0, 3]$

ans: $(0, 0), (\frac{3}{2}, 0), (3, 0)$

12. Find exact value of $\cos(\tan^{-1}(\frac{\sqrt{3}}{3}))$ **ans:** $\frac{\sqrt{3}}{2}$

13. Find exact value of $\cos(\sin^{-1}(\frac{4}{5}))$ **ans:** $\frac{3}{5}$

14. Find exact value of $\cos^{-1} \frac{\sqrt{3}}{2}$ **ans:** $\frac{\pi}{6}$

15. Find horizontal asymptote of $y = \tan^{-1}(x)$ [look this up in your notes]

Section 5

Verify the following identities (# 1 - 3).

1. $\frac{1 - \cos^2 x}{\cos x} = \sin x \tan x$

2. $(\sin x + \cos x)(\csc x - \sec x) = \cot x - \tan x$

3. $\frac{1 + \sin x}{1 - \sin x} = (\tan x + \sec x)^2$

Simplify (# 4 - 6).

4. $\cos\left(\frac{\pi}{2} + x\right)$ **ans:** $-\sin x$

5. $\sin\left(\frac{\pi}{4} - x\right) \cos \frac{\pi}{4} + \cos\left(\frac{\pi}{4} - x\right) \sin \frac{\pi}{4}$

ans: $\cos x$

6. $\sin \alpha = \frac{3}{5}$, $\cos \beta = -\frac{5}{13}$, α and β are in quadrant II; find $\cos(\alpha + \beta)$.

ans: $-\frac{16}{65}$

Verify each identity (#7 -8).

7. $\frac{1 + \cot x}{\cot x - 1} = \frac{1 + \sin 2x}{\cos 2x}$

8. $\cos(x + y) \cos(x - y) = \cos^2 x + \cos^2 y - 1$

Find all solutions of each equation in the interval $[0, 2\pi)$.

9. $\tan x - 2 \tan x \cos x = 0$

ans: $0, \frac{\pi}{3}, \frac{5\pi}{3}$

10. $2 \cos^2 x - \cos x = 1$

ans: $0, \frac{2\pi}{3}, \frac{4\pi}{3}$

11. $2 \sec^3 x + \sec^2 x - 8 \sec x - 4 = 0$

ans: $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Find all solutions of each equation in the interval $[0, 2\pi)$.

12. $\tan^2 x = 3 \sec^2 x - 2$

ans: No solution

13. $\cos^2 x - 3 \sin x + 2 \sin^2 x = 0$

ans: $22.5^\circ, 157.5^\circ$

14. $\cos x + 3 = 0$

ans: No solution

15. $4 \sin^2 x + 2\sqrt{3} \sin x - 2 \sin x - \sqrt{3} = 0$

ans: $30^\circ, 150^\circ, 240^\circ, 300^\circ$

Section 6

Solve each triangle (#1 - 5).

1. $a = 24, b = 32, c = 28$

ans: $C \approx 58^\circ, A \approx 47^\circ, B = 76^\circ$

2. $b = 102, c = 150, A = 82^\circ$

ans: $a \approx 169, B = 37^\circ, C = 61^\circ$

3. $C = 55^\circ, c = 80, b = 110$

ans: No triangle is formed, as $\sin B \approx 1.1263$

4. $B = 25^\circ, C = 40^\circ, c = 40$

ans: $A = 115^\circ, a \approx 56, b \approx 26$

5. $A = 37^\circ, c = 40, a = 28$

ans: $C = 59^\circ, B = 84^\circ, b = 46$ or $C = 121^\circ, B = 22^\circ, b = 17$

6. Find the components of a vector whose initial and terminal points are $P_1(-4,0); P_2(-3,6)$.

ans: $\mathbf{v} = \langle 1, 6 \rangle$

7. Find the magnitude and direction of the vector $\mathbf{v} = \langle -5, 5\sqrt{3} \rangle$

ans: magnitude = 10, direction = 120°

8. Find the magnitude and direction of $z = 2 - 3i$.

ans: $r = \sqrt{13}, \theta \approx 304^\circ$

9. Write the complex number $z = 5(\cos 315^\circ + i \sin 315^\circ)$ in standard form.

ans: $z \approx 3.54 - 3.54i$

10. Find the vertices and asymptotes of the hyperbola $\frac{y^2}{4} - \frac{x^2}{25} = 1$

ans: vertices - $(0, 2), (0, -2)$

asymptotes - $y = \pm \frac{2}{5}x$

11. Find the vertices of the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$

ans: $(-6, 0), (6, 0)$

12. Find the 30th term of the sequence $a_n = (-1)^{n+1} \left(\frac{n}{n+3} \right)$

ans: $-\frac{10}{11}$